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Predictive Models of Large Neutrino Mixing Angles *

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Abstract

Several experimental results could be interpreted as evidence that certain neutrino mixing angles are large, of order unity. However, in the context of grand unified models the neutrino angles come out characteristically to be small, like the KM angles. It is shown how to construct simple grand-unified models in which neutrino angles are not only large but completely predicted with some precision. Six models are presented for illustration.

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1 Introduction

There are hints that some neutrino mixing angles may be large. One interpretation of atmospheric neutrino data[1] suggests that the mixing angle between ν_μ and ν_τ is of order unity.[2] There is also a large-angle solution[3] to the MSW explanation[4] of the solar neutrino problem.[5] However, in unified theories of quark and lepton masses there is a tendency for the leptonic mixing angles, like the quark mixing angles, to come out small. In particular, they tend to come out proportional to powers (either 1 or $\frac{1}{2}$) of the small intergenerational mass ratios.

In a recent paper[6] a general idea was proposed which gives in a simple and natural way small KM angles and large neutrino mixing angles in the context of unified theories. This general idea has the additional virtue of explaining why the hierarchy among the up quarks is larger than that among the down quarks and leptons. In that same paper[6] it was shown that this idea could be combined with the idea of quark and lepton mass-matrix "textures" to give highly predictive schemes in which the full 3×3 unitary mixing matrix of the neutrinos is accurately predicted. In this paper we present a set of five new models which (together with an example given in Ref. 6) realize these ideas, and which give definite and distinguishable predictions for the neutrino mixing angles. These models not only illustrate the possibilities of this approach, but demonstrate that at least within this framework an experimental determination of the neutrino mixing angles can settle the question of the origin of the pattern of light fermion masses.

2 The General Idea

The general idea can be simply explained in the context of SU(5). Consider a model where the fermions are in the representations $(\overline{\mathbf{5}}_i + \mathbf{10}_i + \mathbf{1}_i) + (\overline{\mathbf{10}}'_i + \mathbf{10}'_i)$, where $i = 1, 2, 3$ is a family index. Let the fermion mass terms be

$$\begin{aligned} \mathcal{L}_{\rm mass} \;=\; & \textstyle\sum_{i,j} l_i^{(0)c} (L_0)_{ij} l_j^{(0)} + \sum_{i,j} d_i^{(0)c} (D_0)_{ij} d_j^{(0)} \\ & + \textstyle\sum_{i,j} u_i^{(0)c} (U_0)_{ij} u_j^{(0)} + \sum_{i,j} \nu_i^{(0)c} (N_0)_{ij} \nu_j^0 \\ & + \textstyle\sum_i M_i \overline{\mathbf{10}}'_i \mathbf{10}'_i + \sum_i m_i \overline{\mathbf{10}}'_i \mathbf{10}_i . \end{aligned} \tag{1}$$

The fields $l_i^{(0)c}$, $u_i^{(0)}$, $d_i^{(0)}$, and $u_i^{(0)c}$ belong to the $\mathbf{10}$ of SU(5) denoted $\mathbf{10}_i$. The fields $l_i^{(0)}$, $\nu_i^{(0)}$, and $d_i^{(0)c}$ belong to the $\overline{\mathbf{5}}$ denoted $\overline{\mathbf{5}}_i$. In addition there is a set of vectorlike pairs denoted $\overline{\mathbf{10}}'_i + \mathbf{10}'_i$.

Note that we write the matrices, L_0 etc., so that the left-handed fermions are to the right and the left-handed antifermions are to the left. This will be the convention throughout this paper. The matrices L_0, D_0, U_0, and N_0 do not have to satisfy the minimal SU(5) relations, but will in general come from effective operators that involve the GUT-scale breaking of SU(5). That is why we write these mass terms using $SU(3)\times SU(2)\times U(1)$ representations instead of $SU(5)$ multiplets. We imagine in this paper that these four matrices are constrained by some kind of family symmetry to have a "texture" form. Moreover, we assume that for each of these matrices all the non-zero elements are of the same order of magnitude. That is to say, the matrices L_0, D_0, U_0, and N_0 do not exhibit a significant intergenerational hierarchy.

The intergenerational hierarchies come from the mixing with the $\overline{\mathbf{10}}'_i + \mathbf{10}'_i$ in the following way. As is clear from Eq. (1), the $\overline{\mathbf{10}}'_i$ gets a Dirac

mass, assumed to be superheavy, with the linear combination $(\cos\theta_i \mathbf{10}'_i + \sin\theta_i \mathbf{10}_i) \equiv \mathbf{10}_{\text{heavy},i}$, where $\tan\theta_i = m_i/M_i$. The orthogonal combination $(-\sin\theta_i \mathbf{10}'_i + \cos\theta_i \mathbf{10}_i) \equiv \mathbf{10}_{\text{light},i}$ is light and contains the Weak-scale-mass observable states, u_i, d_i, u_i^c, and l_i^c. Thus, the $\mathbf{10}_i$, which contains the fields $l_i^{(0)c}$, $u_i^{(0)}$, $d_i^{(0)}$, and $u_i^{(0)c}$ appearing in Eq.(1) is related to the true low-mass states by

$$\mathbf{10}_i = \cos\theta_i \mathbf{10}_{\text{light},i} + \sin\theta_i \mathbf{10}_{\text{heavy},i}. \tag{2}$$

That means that we can write the mass matrices of the light quarks and leptons as

$$\begin{array}{rcl} L & = & H\ L_0 \\ D & = & D_0\ H \\ U & = & H\ U_0\ H \\ N & = & N_0, \end{array} \tag{3}$$

where

$$H = \begin{pmatrix} \cos\theta_1 & & \\ & \cos\theta_2 & \\ & & \cos\theta_3 \end{pmatrix} \equiv \begin{pmatrix} \epsilon_1 & & \\ & \epsilon_2 & \\ & & \epsilon_3 \end{pmatrix}. \tag{4}$$

From these equations it is clear that if there is a hierarchy $\epsilon_1 \ll \epsilon_2 \ll \epsilon_3$, that is to say if $M_1/m_1 \ll M_2/m_2 \ll M_3/m_3$, then there will be an intergenerational hierarchy both among the masses of the down quarks and among those of the charged leptons that goes as $\epsilon_1 : \epsilon_2 : \epsilon_3$, and a hierarchy among the up quark masses that goes as $\epsilon_1^2 : \epsilon_2^2 : \epsilon_3^2$. From Eq.(3) one also sees that the mixing angles among the left-handed quarks are of order the hierarchy factors, that is $V_{ij}^{KM} \sim \epsilon_i/\epsilon_j$, $i < j$, while the mixing angles among

the left-handed leptons are of order unity (since no factor of H appears to the right of L_0 and N_0 in Eq. (3)).

To be able to actually predict the neutrino mixing angles from the knowledge we already possess about the quark and lepton masses and the KM angles there must be a symmetry that relates N_0 to L_0, D_0, and U_0. This would suggest enlarging the symmetry group to SO(10). In that case the representations $\overline{\mathbf{5}}_i + \mathbf{10}_i + \mathbf{1}_i$ are unified into $\mathbf{16}_i$, and the $\overline{\mathbf{10}}'_i + \mathbf{10}'_i$ could be unified either into $\mathbf{45}_i$ or into $\overline{\mathbf{16}}'_i + \mathbf{16}'_i$.

Unifying the $\overline{\mathbf{10}}'_i + \mathbf{10}'_i$ into $\mathbf{45}_i$ would mean that only the $\mathbf{10}_i$, and not the $\overline{\mathbf{5}}_i$, mixed with the vectorlike states, since the $\mathbf{45}$ contains $\overline{\mathbf{10}} + \mathbf{10}$ but not $\overline{\mathbf{5}} + \mathbf{5}$. This would lead to the structure shown in Eqs. (1)-(3), and is the situation that is assumed in this paper.

However, the other possibility is also interesting. If the $\overline{\mathbf{10}}'_i + \mathbf{10}'_i$ are contained in $\overline{\mathbf{16}}'_i + \mathbf{16}'_i$, then there are also $\mathbf{5}'_i + \overline{\mathbf{5}}'_i$ with which the $\overline{5}_i$ (that contain the $l_i^{(0)}$, $\nu_i^{(0)}$, and $d_i^{(0)c}$) mix. If, instead of just mass terms like $M_i \overline{\mathbf{16}}'_i \mathbf{16}'_i + m_i \overline{\mathbf{16}}'_i \mathbf{16}_i$ analogous to the terms in Eq. (1), one had as well terms where M_i and m_i were replaced by the VEV of an adjoint Higgs field, $\mathbf{45}_H$, which pointed along the SU(5)-singlet generator of SO(10), then different mixing matrices, H_{10}, $H_{\overline{5}}$, and H_1, would exist for the $\mathbf{10}$, $\overline{\mathbf{5}}$, and $\mathbf{1}$ of SU(5). Then one would have $L = H_{10} L_0 H_{\overline{5}}$, $D = H_{\overline{5}} D_0 H_{10}$, $U = H_{10} U_0 H_{10}$, and $N = H_1 N_0 H_{\overline{5}}$. We shall not explore this possibility in this paper.

Assuming that the $\overline{\mathbf{10}}'_i + \mathbf{10}'_i$ are unified into $\mathbf{45}_i$, the terms involving the vectorlike fermions in Eq.(1) become at the SO(10) level $(\sum_i M_i \mathbf{45}_i \mathbf{45}_i + \sum_i m_i \mathbf{45}_i \mathbf{16}_i \langle \overline{\mathbf{16}}_H \rangle)$. These terms will also generate GUT-scale right-handed neutrino masses, since the $\mathbf{45}_i$ contains singlets, $\mathbf{1}'_i$, which will mix with the

singlets $\mathbf{1}_i$. It is easy to see by integrating out the superheavy singlets $\mathbf{1}'_i$ and $\mathbf{1}_i$ that the Majorana mass matrix of the light, left-handed neutrinos takes the form

$$(M_\nu)_{ij} = \frac{4}{5}(N_0^T)_{ik} m_k^{-1} M_k m_k^{-1} (N_0)_{kj}, \tag{5}$$

or

$$M_\nu = \frac{4}{5} N_0^T \tilde{H} M^{-1} \tilde{H} N_0, \tag{6}$$

where $\tilde{H} \equiv \mathrm{diag}(\cot\theta_1, \cot\theta_2, \cot\theta_3) \simeq H = \mathrm{diag}(\epsilon_1, \epsilon_2, \epsilon_3)$ and the $\frac{4}{5}$ is an SO(10) Clebsch. $\tilde{H}$ has a hierarchy similar to that of H.

From the forms in Eqs. (3) and (4) it is straightforward to derive explicit expressions for the mass ratios of the charged quarks and leptons and for the KM angles; and from Eq. (6) one can in the same way derive expressions for the neutrino mixing angles as we shall see.

From Eqs. (3) and (4) it is apparent that the elements of U have a hierarchy $U_{ij} \propto \epsilon_i \epsilon_j$. That is to say, there is a hierarchy in both the rows and columns of U. Therefore,

$$m_c/m_t \cong (\epsilon_2/\epsilon_3)^2 \, \frac{\det_{23} U_0}{(U_0)_{33}^2}, \tag{7}$$

$$m_u/m_t \cong (\epsilon_1/\epsilon_3)^2 \, \frac{\det U_0}{(U_0)_{33} \det_{23} U_0}. \tag{8}$$

The down-quark matrix, $D = D_0 H$, has a hierarchy among its columns, but not among its rows. Thus it is convenient to define the column vectors $(\vec{D}_j)_i = (D_0)_{ij}$. Then it is straightforward to show that

$$m_s/m_b \cong (\epsilon_2/\epsilon_3)\frac{\left|\vec{D}_2 \times \vec{D}_3\right|}{\left|\vec{D}_3\right|^2}, \tag{9}$$

$$m_d/m_b \cong (\epsilon_1/\epsilon_3)\frac{\left|\vec{D}_1 \cdot \vec{D}_2 \times \vec{D}_3\right|\left|\vec{D}_3\right|}{\left|\vec{D}_2 \times \vec{D}_3\right|^2}. \tag{10}$$

Since both U and D have hierarchies among their *columns*, the rotations among the *left-handed* u_i and d_i required to diagonalize the mass matrices will be small, proportional to hierarchy factors ϵ_i/ϵ_j. One can write down the leading order (in ϵ_i/ϵ_j, $i < j$) expressions for the Kobayashi-Maskawa angles in a simple form.

$$\begin{aligned}
V_{cb} &\cong \left(\tfrac{\epsilon_2}{\epsilon_3}\right)\left[\tfrac{\vec{D}_2\cdot\vec{D}_3}{(\vec{D}_3)^2} - \tfrac{U_{0,32}}{U_{0,33}}\right], \\
V_{us} &\cong \left(\tfrac{\epsilon_1}{\epsilon_2}\right)\left[\tfrac{\vec{D}_1\cdot\vec{D}_2 - \vec{D}_1\cdot\hat{D}_3\vec{D}_2\cdot\hat{D}_3}{|\vec{D}_2\times\hat{D}_3|^2} - \tfrac{U_{0,33}U_{0,21}-U_{0,31}U_{0,23}}{\det_{23}U_0}\right], \\
V_{ub} &\cong \left(\tfrac{\epsilon_1}{\epsilon_3}\right)\left[\tfrac{\vec{D}_1\cdot\vec{D}_3}{(\vec{D}_3)^2} - \tfrac{U_{0,31}}{U_{0,33}} + \left(\tfrac{U_{0,33}U_{0,21}-U_{0,31}U_{0,23}}{\det_{23}U_0}\right)\left(\tfrac{\vec{D}_2\cdot\vec{D}_3}{(\vec{D}_3)^2} - \tfrac{U_{0,32}}{U_{0,33}}\right)\right].
\end{aligned} \tag{11}$$

Note that $V_{ub} \sim V_{us}V_{cb}$.

The expressions for the mass ratios of the charged leptons are similar in form to those of the down quarks, except that $L = HL_0$ has a hierarchy among its *rows* and not its columns. Thus it is convenient to define the row vectors $(\vec{L}_i)_j \equiv (L_0)_{ij}$. In terms of these

$$m_\mu/m_\tau \cong (\epsilon_2/\epsilon_3)\frac{\left|\vec{L}_2 \times \vec{L}_3\right|}{\left|\vec{L}_3\right|^2}, \tag{12}$$

$$m_e/m_\tau \cong (\epsilon_1/\epsilon_3)\frac{\left|\vec{L}_1\cdot\vec{L}_2\times\vec{L}_3\right|\left|\vec{L}_3\right|}{\left|\vec{L}_2\times\vec{L}_3\right|^2}. \tag{13}$$

In discussing the neutrino mixing angles let us assume for the moment that the masses M_i in Eq. (1) are all of the same order, so that the hierarchy among the $\epsilon_i \equiv \cos\theta_i = M_i/\sqrt{m_i^2+M_i^2}$ is due to a hierarchy among the m_i. Then it is apparent from Eqs. (5) and (6) that one has effectively as a neutrino Dirac mass matrix $N_{\nu,\mathrm{eff}} \equiv \tilde{H}N_0$. This, like $L = HL_0$ has a hierarchy among its rows but not among its columns. Therefore, the leptonic analogue of the KM matrix has mixing angles of order unity, and to leading order the small parameters ϵ_i/ϵ_j, $i<j$, do not enter. It is straightforward to show that

$$V_{\mathrm{lepton}} = V_N^\dagger V_L, \tag{14}$$

where

$$V_L \cong \left(\frac{\vec{L}_2\times\hat{L}_3}{|\vec{L}_2\times\hat{L}_3|}, \frac{\vec{L}_2-\vec{L}_2\cdot\hat{L}_3\hat{L}_3}{|\vec{L}_2\times\hat{L}_3|}, \hat{L}_3\right), \tag{15}$$

and

$$V_N \cong \left(\frac{\vec{N}_2\times\hat{N}_3}{|\vec{N}_2\times\hat{N}_3|}, \frac{\vec{N}_2-\vec{N}_2\cdot\hat{N}_3\hat{N}_3}{|\vec{N}_2\times\hat{N}_3|}, \hat{N}_3\right). \tag{16}$$

If, indeed, the M_i are all of the same order, with the hierarchy being among the m_i, then the rows of $N_{\nu,\mathrm{eff}} = \tilde{H}N_0$ have a hierarchy of order $\epsilon_1 : \epsilon_2 : \epsilon_3$. In that case the corrections to Eq. (14) are easily shown to be $\delta(V_{\mathrm{lepton}})_{ij} \sim (\epsilon_i/\epsilon_j)^2$, $i<j$. On the other hand, it could be that the m_i

are all of the same order, with the hierarchy being among the M_i. In that case one can write Eq. (4) more usefully as $M_\nu = \frac{4}{5} N_0^T \tilde{H}^{\frac{1}{2}} m^{-1} \tilde{H}^{\frac{1}{2}} N_0$. From this it is evident that the corrections to the expression for $V_{\rm lepton}$ given in Eq. (14) are of order $\delta(V_{\rm lepton})_{ij} \sim \epsilon_i/\epsilon_j$. In either case, the corrections, as we shall see, are small enough in realistic cases to mean that the predictions of particular models are sufficiently sharp to allow them to be distinguished.

3 Texture Models

(a) An example: Model Aa

We will construct models in which the matrices L_0, D_0, U_0, and N_0 have a common "texture" form. An example, which was given in Ref. 6, is the following:

$$L_0 = \begin{bmatrix} & -3D & \\ D & -C & \\ & B/2 & A \end{bmatrix}, \tag{17}$$

$$D_0 = \begin{bmatrix} & D & \\ -3D & C/3 & \\ & B/2 & A \end{bmatrix}, \tag{18}$$

$$U_0 = \begin{bmatrix} & D & \\ D & C/3 & \\ & -B/2 & A \end{bmatrix} \tan\beta, \tag{19}$$

$$N_0 = \begin{bmatrix} & -3D & \\ 5D & -C & \\ & -B/2 & A \end{bmatrix} \tan\beta, \tag{20}$$

with $B/A = 0.4$, $C/A = 0.75$, $D/A = 0.06$, $\epsilon_2/\epsilon_3 = 0.08$, and $\epsilon_1/\epsilon_3 = 0.02$. This gives the following fit to the quark and lepton masses and mixings:

$m_\tau/m_b = 1.02$, $m_\mu/m_s \cong 3.0$, $m_e/m_d \cong 0.33$, $m_\mu/m_\tau \cong 0.06$, $m_e/m_\mu \cong 5 \times 10^{-3}$, $m_c/m_t \cong 1.6 \times 10^{-3}$, $m_u/m_c \cong 3.5 \times 10^{-3}$, $V_{us} \cong 0.22$, $V_{ub} \cong 0.002$, and $V_{cb} \cong 0.03$. (These quantities are all defined at the unification scale.)

With these values of the parameters, one finds, using Eqs. (14)-(16) that

$$V_{\rm lepton}^{(Aa)} = \left(\begin{array}{ccc} 0.95 & 0.3 & -0.088 \\ -0.3 & 0.87 & -0.39 \\ 0.032 & 0.4 & 0.92 \end{array} \right). \quad (21)$$

The superscript (Aa) is the name we give to this particular model in this paper, for reasons that will become apparent later. It should be noted that all four matrices in Eqs. (17)-(20) have the same form, which can be written

$$F_0 \propto \left(\begin{array}{ccc} 0 & D\; X[f] & 0 \\ D\; X[f^c] & C\; (B-L)[f^c] & B\; I_{3R}[f] \\ 0 & B\; I_{3R}[f^c] & A \end{array} \right), \quad (22)$$

with $F = L$, D, U, or N. The quantities $B-L$, I_{3R}, and X are just generators of SO(10). $B-L$ and I_{3R} (the third generator of $SU(2)_R$) are conventionally normalized. X is the SU(5)-singlet generator that is normalized so that the **10**, $\overline{\mathbf{5}}$, and **1** contained in the **16** have the charges 1, -3, and 5 respectively. Sometimes we shall use generators, $\overline{Q}$, consistently normalized so that $\mathrm{tr}|_{16}\overline{Q}^2 = 1$. Then $\overline{B-L} = \frac{\sqrt{3}}{4}(B-L)$, $\overline{I_{3R}} = \frac{1}{\sqrt{2}} I_{3R}$, and $\overline{X} = \frac{1}{4\sqrt{5}} X$. Writing Eq. (22) in terms of those normalized generators

$$F_0 = \left(\begin{array}{ccc} 0 & \overline{D}\; \overline{X}[f] & 0 \\ \overline{D}\; \overline{X}[f^c] & \overline{C}\; (\overline{B-L})[f^c] & \overline{B}\; \overline{I_{3R}}[f] \\ 0 & \overline{B}\; \overline{I_{3R}}[f^c] & \overline{A} \end{array} \right). \quad (23)$$

One has then that $\overline{B}/\overline{A} = \sqrt{2} B/A = 0.566$, $\overline{C}/\overline{A} = 1.73$, and $\overline{D}/\overline{A} = 0.537$. Since we are attempting to explain the intergenerational hierarchies by the

mixing parameters ϵ_2/ϵ_3 and ϵ_1/ϵ_3, it is most natural if the ratios of these barred parameters are of order unity, as indeed we see that they are in this model. One can regard this as an encouraging success of this approach.

Another success of this approach is the fact that the same form can be used in all four matrices, L_0, D_0, U_0, and N_0. In usual texture models, using the same form for U and D either gives $V_{cb} \cong 0$ (if U_{32} and $D_{32} = 0$), or $V_{cb} \sim \sqrt{m_s/m_b}$ (if U_{32} and $D_{32} \neq 0$), which is much too large. Here, even with the same form for D_0 and U_0 (up to the group theory factors), $V_{cb} \sim \epsilon_2/\epsilon_3 \sim m_s/m_b$, which is of the correct order.

The generators of SO(10) can be introduced into the form F_0 simply through higher-dimension effective operators obtained from integrating out vectorlike fermion representations. Consider the following set of terms

$$\mathcal{L}' = a\overline{\mathbf{16}}\Omega_{\tilde{Q}}\mathbf{16} + b\overline{\mathbf{16}}\Omega_Q\mathbf{16}_i + c\mathbf{16}\mathbf{16}_j\mathbf{10}_H. \tag{24}$$

here i and j are not dummy indices but are particular values of the indices. Ω_Q is either an adjoint ($\mathbf{45}$) of Higgs fields, whose VEV is proportional to the SO(10) generator Q, or it is an explicit mass or singlet Higgs, in which case Q is just the identity. The same possibilities exist for $\Omega_{\tilde{Q}}$. Both Ω_Q and $\Omega_{\tilde{Q}}$ are taken to be of the GUT scale. It is easy to see that if one integrates out the superheavy fermion $\overline{\mathbf{16}}$ and its superheavy partner $\mathbf{16}_{\text{heavy}} \propto a\langle\Omega_{\tilde{Q}}\rangle\mathbf{16} + b\langle\Omega_Q\rangle\mathbf{16}_i$ one obtains the following effective operator

$$\mathcal{O} = c\frac{b\langle\Omega_{Q(\mathbf{16}_i)}\rangle}{\sqrt{|b\langle\Omega_{Q(\mathbf{16}_i)}\rangle|^2 + |a\langle\Omega_{\tilde{Q}(\mathbf{16}_i)}\rangle|^2}}\mathbf{16}_i\mathbf{16}_j\mathbf{10}_H. \tag{25}$$

Here $Q(\mathbf{16}_i)$ is the value of Q acting on the appropriate component of the $\mathbf{16}_i$.

Let us assume that $|b\langle\Omega_Q\rangle|^2 \ll |a\langle\Omega_{\tilde{Q}}\rangle|^2$. Then the operator is approximately

$$\mathcal{O} \propto \frac{Q(\mathbf{16}_i)}{\tilde{Q}(\mathbf{16}_i)} \mathbf{16}_i \mathbf{16}_j \mathbf{10}_H. \tag{26}$$

Consider the contributions of this operator to the matrix F_0. There are two contributions.

$$\mathcal{O}_f = \frac{Q(f^c)}{\tilde{Q}(f^c)} f_i^c f_j v^{(f)} + \frac{Q(f)}{\tilde{Q}(f)} f_i f_j^c v^{(f)}. \tag{27}$$

If $i \neq j$ one has the combination of generators $Q(f^c)/\tilde{Q}(f^c)$ appearing in the ij element of F_0, and $Q(f)/\tilde{Q}(f)$ appearing in the ji element. For example, in the model described above (the (Aa) model, cf. Eq. (23)) one can get the 23 and 32 elements of the right form by taking $i = 3$, $j = 2$, $Q = I_{3R}$, $\tilde{Q} = 1$; and one can get the 12 and 21 elements by taking $i = 2$, $j = 1$, $Q = X$, and $\tilde{Q} = 1$.

For $i = j$, the operator in Eq. (26) leads to the combination of generators $[Q(f^c)/\tilde{Q}(f^c) + Q(f)/\tilde{Q}(f)]$ appearing in the ii element of F_0. In the example model, one gets the 33 element in a trivial way by taking $i = j = 3$ and $Q = \tilde{Q} = 1$. The 22 element of that model requires more discussion. For $i = j = 2$ take $Q = B - L$ and $\tilde{Q}$ to be a linear combination of I_{3R} and 1. (That is, $\Omega_{\tilde{Q}}$ is a linear combination of an adjoint with VEV proportional to I_{3R} and an explicit mass.) In particular, take

$$\frac{Q[f^c]}{\tilde{Q}[f^c]} + \frac{Q[f]}{\tilde{Q}[f]} = \frac{(B-L)[f^c]}{\eta^{-1} I_{3R}[f^c] + 1} + \frac{(B-L)[f]}{\eta^{-1} I_{3R}[f] + 1}, \tag{28}$$

with $\eta \ll 1$. Since the left-handed fermions have $I_{3R} = 0$, the second term is just $(B-L)[f]$. But $I_{3R}[f^c] = \pm\frac{1}{2}$, so the first term is $\pm 2\eta (B-L)[f^c] \ll 1$.

For small η we can therefore neglect the first term and the combination of generators is approximately $(B-L)[f] = -(B-L)[f^c]$.

(b) A Second Example: Model Bb

Our second example is given by

$$L_0 = \begin{bmatrix} E/2 & & \\ D/2 & C/2 & \\ & B/2 & A \end{bmatrix}, \tag{29}$$

$$D_0 = \begin{bmatrix} -3E/2 & & \\ D/2 & -C/6 & \\ & -3B/2 & A \end{bmatrix}, \tag{30}$$

$$U_0 = \begin{bmatrix} 3E/2 & & \\ -D/2 & -C/2 & \\ & 3B/2 & A \end{bmatrix} \tan\beta, \tag{31}$$

$$N_0 = \begin{bmatrix} -E/2 & & \\ -D/2 & -C/10 & \\ & -B/2 & A \end{bmatrix} \tan\beta, \tag{32}$$

with $B/A = 0.36$, $C/A = 4.6$, $D/A = 1.15$, $E/A = 0.10$, $\epsilon_2/\epsilon_3 = 0.027$, $\epsilon_1/\epsilon_3 = 0.0062$. This gives the same values for the mass ratios and KM angles as model (Aa), except that the value of $V_{ub}/(V_{us}V_{cb})$ comes out to be about 0.25 instead of 0.3. The four mass matrices have the common form

$$F_0 = \begin{pmatrix} E\left(\frac{I_{3R}}{B-L}\right)[f^c] & D\ I_{3R}[f] & \\ D\ I_{3R}[f^c] & C\ \left(\frac{I_{3R}}{X}\right)[f^c] & B\ \left(\frac{I_{3R}}{B-L}\right)[f] \\ & B\ \left(\frac{I_{3R}}{B-L}\right)[f^c] & A \end{pmatrix}. \tag{33}$$

If we use the consistently normalized SO(10) generators, then we find $\overline{B}/\overline{A} = 0.22$, $\overline{C}/\overline{A} = 0.727$, $\overline{D}/\overline{A} = 1.63$, $\overline{E}/\overline{A} = 0.061$. Except for the last quantity all the parameter ratios are of order unity.

The neutrino-mixing matrix obtained from Eqs. (14) - (16) is

$$V_{\text{lepton}}^{(Bb)} = \begin{pmatrix} -0.78 & 0.56 & 0.27 \\ -0.62 & -0.76 & -0.215 \\ 0.086 & -0.34 & 0.94 \end{pmatrix}. \tag{34}$$

In both the models discussed so far $U_{0,31} = 0 = \vec{D}_1 \cdot \vec{D}_3$. In this case Eqs. (11) simplify to give

$$\frac{V_{ub}}{V_{us}V_{cb}} \cong \left[\left(\frac{\vec{D}_1 \cdot \vec{D}_2}{|\vec{D}_2 \cdot \hat{D}_3|^2} \right) / \left(\frac{U_{0,33}U_{0,21}}{\det_{23} U_0} \right) - 1 \right]^{-1}. \tag{35}$$

(c) Six Models and Predictions for Neutrino Mixing

All the models presented here have the following general form

$$F_0 = \begin{pmatrix} c_{11}^F E & c_{12}^F D & 0 \\ c_{21}^F D & c_{22}^F C & c_{23}^F B \\ 0 & c_{32}^F B & c_{33}^F A \end{pmatrix}, \tag{36}$$

where, for each ij, $i \neq j$, there is a pair of generators Q, $\tilde{Q}$, such that $c_{ij}^F = Q[f^c]/\tilde{Q}[f^c]$, $c_{ji}^F = Q[f]/\tilde{Q}[f]$, and where for each ii there is a pair of generators Q, $\tilde{Q}$ such that $c_{ii}^F = (Q[f^c]/\tilde{Q}[f^c] + Q[f]/\tilde{Q}[f])$. In Table I we present for each of the six models the group-theoretical factors that appear in each of the entries of F_0. In Table II are given the numerical values of the parameters of the models that give good fits to the observed quark and lepton masses and KM angles. In Table III are given the neutrino-mixing matrices that are predicted in each model from Eqs. (14)-(16), as well as the values of $V_{ub}/(V_{us}V_{cb})$ that are predicted from Eqs. (11).

The reason for the names we have given the models can be seen from Table I. The models with the same capital letter have the same form in the

2-3 block. This means that they get the Georgi-Jarlskog ratio $m_\mu/m_s \cong 3$ in the same way. There are three such forms used, which we call A, B, and C. Similarly, models with the same lower-case letter have the same form in the 1-2 block (more precisely, in the 11, 12, and 21 elements). There are three such forms used which we have called a, b, and c. These forms are arranged to give the other Georgi-Jarlskog factor $m_e/m_d \cong \frac{1}{3}$.

From Table II we see that the values of the ratios $\overline{B}/\overline{A}$, $\overline{C}/\overline{A}$, and $\overline{D}/\overline{A}$ are of order unity, as is natural in this framework where the intergenerational hierarchies come from the ratios ϵ_i/ϵ_j. In particular, the 18 entries in Table II that give these three ratios for the six models are all between $\frac{1}{10}$ and 10. Indeed, 12 of these 18 numbers are between $\frac{1}{2}$ and 2. The values of $\overline{E}/\overline{A}$ are somewhat smaller for the five models which have this parameter, ranging from 0.058 to 0.331, and being typically about $\frac{1}{10}$.

It should be noted that the signs of the entries in $V_{\rm lepton}$ shown in Table III are not individually of absolute significance. First of all, a change in the sign of ϵ_i/ϵ_j gives a change in the signs of certain fermion mass ratios and KM angles. Since these are not known, one can get equally good fits to the known data by assuming lepton mass ratios of various signs. Thus, one can change the sign of any of the left or right-handed lepton mass eigenstates and have essentially the same fit. Therefore in $V_{\rm lepton}$ the sign of any row or column can be changed and still correspond to a model which fits the known data.

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Table I: The combinations of generators appearing in each entry of the fermion mass matrix F_0 for each of the six models. These generators act on the left-handed f^c for entry ij $(i \neq j)$, on the left-handed f for entry ji, and on the left-handed f for diagonal entries ii. For the 22 column the entries "$(B-L)_f$" stand for the expression $(B-L)/(\eta^{-1}I_{3R}+1)$ where η is a small parameter. In model Bc, the fourth column entry (distinguished by a (**)) corresponds to $ij = 12$ and not $ij = 21$ as for the other models.

Entry $ij \rightarrow$ / $\downarrow$ Model	33	32	22	21	11
Aa	1	$I_{3R}/1$	$(B-L)_f$	$X/1$	$-$
Ab	1	$I_{3R}/1$	$(B-L)_f$	$I_{3R}/1$	$I_{3R}/(B-L)$
Ac	1	$I_{3R}/1$	$(B-L)_f$	$X/1$	$I_{3R}/(B-L)$
Bb	1	$I_{3R}/(B-L)$	I_{3R}/X	$I_{3R}/1$	$I_{3R}/(B-L)$
Bc	1	$I_{3R}/(B-L)$	I_{3R}/X	$1/X$ (**)	$I_{3R}/(B-L)$
Cb	$1/X$	$(B-L)/1$	1	$I_{3R}/1$	$I_{3R}/(B-L)$

Table II: The values of the parameters that give a good fit to the known quark and lepton masses and the KM angles for each of the six models. (The barred quantities are the coefficients of the SO(10)-consistently normalized generators.)

Model	B/A ($\overline{B}/\overline{A}$)	C/A ($\overline{C}/\overline{A}$)	D/A ($\overline{D}/\overline{A}$)	E/A ($\overline{E}/\overline{A}$)	ϵ_2/ϵ_3	ϵ_1/ϵ_3
Aa	0.4 (0.566)	0.75 (1.73)	0.06 (0.537)	$-$	0.08	0.02
Ab	0.4 (0.566)	0.75 (1.73)	-0.66 (-0.933)	0.094 (0.058)	0.08	0.0064
Ac	0.4 (0.566)	0.75 (1.73)	0.3 (2.68)	0.54 (0.331)	0.08	0.0035
Bb	0.36 (0.22)	4.6 (0.727)	1.15 (1.63)	0.10 (0.061)	0.027	0.0062
Bc	0.36 (0.22)	4.6 (0.727)	-1.31 (-0.147)	-0.344 (-0.205)	0.027	0.005
Cb	-0.3 (-6.2)	0.033 (0.30)	0.147 (1.85)	0.025 (0.139)	0.28	0.0157

Table III: The predicted values of the leptonic mixing angles and of V_{ub} in each of the six models.

Model	V_{lepton}	$V_{ub}/(\sin\theta_c V_{cb})$
Aa	$\begin{pmatrix} 0.95 & 0.3 & -0.088 \\ -0.3 & 0.87 & -0.39 \\ 0.032 & 0.4 & 0.92 \end{pmatrix}$	0.3
Ab	$\begin{pmatrix} 0.68 & 0.72 & -0.157 \\ -0.72 & 0.60 & -0.35 \\ -0.157 & 0.35 & 0.92 \end{pmatrix}$	0.5
Ac	$\begin{pmatrix} 0.72 & 0.60 & -0.345 \\ -0.68 & 0.72 & -0.17 \\ 0.145 & 0.355 & 0.92 \end{pmatrix}$	0.25
Bb	$\begin{pmatrix} -0.78 & 0.56 & 0.27 \\ -0.62 & -0.76 & -0.215 \\ 0.086 & -0.34 & 0.94 \end{pmatrix}$	0.25
Bc	$\begin{pmatrix} -0.472 & -0.840 & -0.268 \\ 0.878 & -0.418 & -0.229 \\ 0.081 & 0.344 & 0.936 \end{pmatrix}$	0.56
Cb	$\begin{pmatrix} 0.80 & 0.58 & 0.163 \\ -0.53 & 0.54 & 0.645 \\ 0.29 & -0.604 & 0.75 \end{pmatrix}$	0.56